

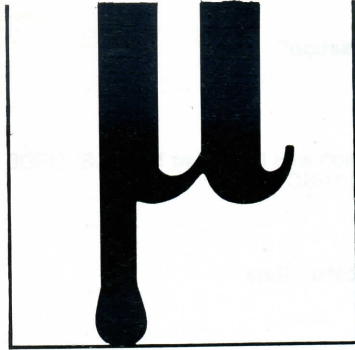
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EXPLICIT

METHODS

of OPTIMIZATION

gauthier-villars



EXPLICIT METHODS

OF OPTIMIZATION

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