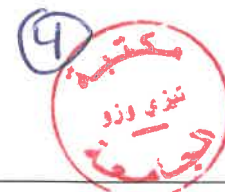
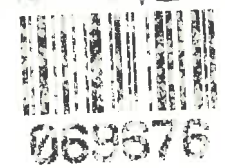




CDD 005.134 076 116

069676



LMD/Engineers

IF 1449

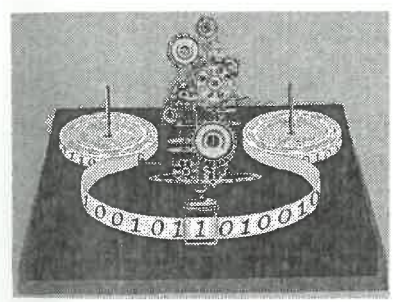
Formal Language Theory and Automata

A Gentle Introduction

Course, exercises and corrected exams

Elkamel MERAH

Abbes Laghrour University, Khenchela Algeria



2nd Year Mathematics-Computing (MI)
2nd Year Computer Science Preparatory Schools (ESI)
Masters and Engineers in Computer Science

© Copyright International Blue Pages



Contents

Preface	iii
Foreword	v
Dedication	ix
1. Languages	1
1. Symbol	2
2. Alphabet	2
3. Word	3
3.1. Basic operations on words	3
4. Languages	8
4.1. Operations on languages	9
5. Three fundamental questions	12
6. Exercises with solutions	13
2. Formal Grammars	17
1. Introduction	18
2. Formal Grammar	19
3. Derivation	22
4. Language generated by a grammar	24
4.1. Equivalence of two grammars	25
5. The Chomsky Hierarchy	26
5.1. General Grammar	26
5.2. Type 1 Grammar	27
5.3. Context-free grammar	29
5.4. Regular Grammar	30
5.5. The empty word	31
6. Concept of automaton	32

2.	Finite automata	47
2.1.	Deterministic finite Automata	47
2.1.1.	Configuration and derivation	47
2.1.2.	Acceptance of a language by finite automata	47
2.1.3.	Representations of a finite automaton	47
2.1.4.	Equivalent automata and Complete automata	50
2.2.	Non-deterministic finite automata	50
2.3.	Conversion of a N DFA to a DFA	50
2.3.1.	Elimination of ϵ -Transitions	60
2.4.	Minimisation of finite automata	60
2.4.1.	Indistinguishability	60
2.4.2.	Nerode congruence	60
2.4.3.	Equivalence class	70
2.4.4.	Quotient automaton	70
2.4.5.	Congruence for words	70
2.4.6.	Moore Algorithm	70
3.	Finite automata and regular grammars	70
3.1.	Regular grammar from finite automaton	70
3.2.	Finite automaton from regular grammar	70
4.	Properties of regular languages	70
4.1.	Ensemblistic properties of regular languages	80
4.2.	Pumping Lemma for regular languages	80
4.3.	Decision problems	90
5.	Regular expressions	90
5.1.	Definitions	90
5.2.	Algebraic properties	90
6.	Finite automata and regular expressions	90
6.1.	Finite automaton from a regular expression	90
6.2.	Regular expression from a finite automaton	100

4.	Context-free languages	121
1.	Context-free grammars	123
1.1.	Derivations	125
1.2.	Derivation tree	127
1.3.	Ambiguity	129
2.	Properties of context-free languages	138
2.1.	Set properties of context-free languages	138
2.2.	The Pumping Lemma for context-free languages	140
2.3.	Decision problems	141
3.	Transforming context-free grammars	142
3.1.	Reduced grammars	142
3.1.1.	Removal of non-productive symbols	143
3.1.2.	Removal of unreachable symbols	145
3.2.	Proper grammars	147
3.2.1.	Removal of ϵ -rules	148
3.2.2.	Removal of unit rules	151
3.3.	Left recursive grammars	153
3.4.	Chomsky Normal Form	158
3.5.	Greibach Normal Form	161
4.	Pushdown automata	163
4.1.	Configuration and computation of a pushdown automaton	166
4.2.	Acceptance of a language by a pushdown automaton	169
4.3.	Determinism versus non-determinism	172
5.	Pushdown automata and context-free grammars	176
5.1.	Pushdown automaton from a context-free grammar	176
6.	Exercises with solutions	179
5.	Recursively enumerable languages	183
1.	General Grammars	185
2.	Turing Machine	188
2.1.	Configuration and Turing Machine computation	191
2.2.	Acceptance of a language by a Turing Machine	194

4.	Determinism versus non-determinism	19
5.	Turing machine as a calculator	19
6.	Exercises with solutions	20
Appendix		20
A.	Solutions to the exercises	20
A.1.	Chapter 1 solutions.	20
A.2.	Chapter 2 solutions.	21
A.3.	Chapter 3 solutions.	21
A.4.	Chapter 4 solutions.	22
A.5.	Chapter 5 solutions.	23
B.	Final exams	23
B.1.	Final exam n°1.	23
B.2.	Final exam n°2.	24
B.3.	Final exam n°3.	24
B.4.	Final exam n°4.	24
B.5.	Final exam n°5.	24
C.	Final exams correction	24
C.1.	Final exam n°1 correction	24
C.2.	Final exam n°2 correction	25
C.3.	Final exam n°3 correction	25
C.4.	Final exam n°4 correction	25
C.5.	Final exam n°5 correction	26
D.	Supplementary exercises without answers	26
D.1.	Chapter 1 exercises.	26
D.2.	Chapter 2 exercises.	26
D.3.	Chapter 3 exercises.	26
D.4.	Chapter 4 exercises.	27
D.5.	Chapter 5 exercises.	27

Bibliography

Index

Languages

"No one shall expel us from the paradise
which Cantor has created for us."

—David Hilbert
Über das Unendliche,
Mathematische Annalen, 1926

