Optimal Decision Making in Operations Research and Statistics

Methodologies and Applications

Editors

Irfan Ali, Leopoldo Eduardo Cárdenas-Barrón, Aquil Ahmed and **Ali Akbar Shaikh**

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Editors

Irfan Ali
Department of Statistics & Operations Research
Aligarh Muslim University, Aligarh, India

Leopoldo Eduardo Cárdenas-Barrón
Department of Industrial and Systems Engineering
School of Engineering and Sciences
Technológico de Monterrey, México

Aquil Ahmed
Department of Statistics & Operations Research
Aligarh Muslim University, Aligarh, India

Ali Akbar Shaikh
Department of Mathematics
The University of Burdwan, Burdwan, India

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Preface

Operations Research (OR) has become a powerful technique for optimal decision-making. New techniques and sophisticated analysis tools are required to resolve the challenges arising from modern problems. It leads to the emergence of OR for efficiently determining optimal solutions to problems of real world. Although there are many types of conceivable problems, OR practitioners and researchers have found several problems in different circumstances. Thus, a challenge problem may be in the manufacturing industry area while another may be in the service sector. However, their essential features are the same. Thus, it is possible to describe these problems by naming the general categories into which they fall irrespective of their physical descriptions. A common analytical technique can be used to find the optimal solution to problems belonging to the same general category. In this direction,OR helps make better decision and solve problems in the real world. It uses mathematical relations, statistical computations, engineering techniques, economics and management methodologies to know the consequences of deciding for any possible alternative actions.

The decision-making techniques can be used in industries and services for making business decisions under risk and uncertainty. Furthermore, the decision-making techniques are also applied successfully to almost every possible sphere of human activity. Moreover, decision-making techniques are widely applied in different fields, ranging from almost every branch of science, engineering, industrial management, management planning, medical sciences, social sciences and economics, among others.

The book "Optimal Decision Making in Operations Research & Statistics: Methodologies and Applications" has been written by unified authors with a diverse background expertise from the faculties of Operations Research, Management, Applied Statistics and Mathematics. The contributed chapters are based on the vast research experiences of the authors in real-world decision-making problems.

The book is on the recent developments and contributions in optimal decision-making using optimization and statistical techniques. Mathematical modelling of cost-effective management policies are also part of the book.

The book presents challenging and practical real-world applications based on decision-making problems in various fields. The modelling and solution procedures of such real-world problems are provided concisely. This book provides readers a valuable compendium of several decision-making problems as a reference for this field's researchers and industrial practitioners. After reading this book, the readers will understand the formulations of decision-making problems and their solution procedures using appropriate optimization and statistical techniques.

This book broadly covers applications of applied statistics and optimization techniques in decision making in the various areas such as—estimation, control charts, econometric, regression, sampling, stochastic modelling, inventory control and management, transportation problem and optimization.

Finally, this book benefits the teachers, students, researchers, and industrialists working in material science, especially Operations Research and Applied Statistics, as a valuable reference handbook for teaching, learning, and research.

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Chapter 1

A New Version of the Generalized Rayleigh Distribution with Copula, Properties, Applications and Different Methods of Estimation

M Masoom Ali[1], ***Haitham M Yousof***[2,*] **and** ***Mohamed Ibrahim***[3]

1. Introduction

A random variable (RV) Z is said to have the generalized Rayleigh (GR) distribution if its probability density function (PDF) and hence the CDF are given by

$$\pi_{\beta}(z) = 2\beta z e^{-z^2}(1-e^{-z^2})^{\beta-1}, \tag{1}$$

and

$$H_{\beta}(z) = (1-e^{-z^2})^{\beta}, \tag{2}$$

respectively, for $z > 0$, $\beta > 0$. Alizadeh et al. (2016) generalized the Odd G (O-G) family and the proportional reversed hazard rate family (PRHR) by proposing a new broader family called the Odd Burr (OB) family. The CDF of the OB family is given by

$$F_{\delta,\theta,\underline{\xi}}(z) = 1 - \frac{\overline{H}_{\underline{\xi}}(z)^{\delta\theta}}{\left[\overline{H}_{\underline{\xi}}(z)^{\delta} + H_{\underline{\xi}}(z)^{\delta}\right]^{\theta}}, \tag{3}$$

where $\overline{H}_{\underline{\xi}}(z) = 1 - H_{\underline{\xi}}(z)$. The PDF corresponding to (3) is given by

$$f_{\delta,\theta,\underline{\xi}}(z) = \frac{\delta\theta\pi_{\underline{\xi}}(z)H_{\underline{\xi}}(z)^{\delta-1}\overline{H}_{\underline{\xi}}(z)^{\delta\theta-1}}{\left[\overline{H}_{\underline{\xi}}(z)^{\delta} + H_{\underline{\xi}}(z)^{\delta}\right]^{1+\theta}}. \tag{4}$$

For $\theta = 1$, the OB-G family reduces to O-G family (see Gleaton and Lynch (2006)). For $\delta = 1$, the OB-G family reduces to the PRHR (see Gupta and Gupta (2007)).

In this paper, we propose and study a new version of the GR called the Odd Burr GR (OBGR) model. Some of its properties are derived and numerically analyzed. The usefulness and flexibility of the OBGR distribution is illustrated by means of a real data set related to failure times. Many bivariate and multivariate type distributions are derived based on Farlie Gumbel Morgenstern (FGM) Copula, modified FGM Copula, Clayton Copula and Renyi's entropy Copula. We briefly describe and consider different estimation methods namely, the maximum likelihood estimation (MLE), Cramér-von-Mises estimation (CVM), ordinary least square estimation (OLS), weighted least square estimation (WLSE), Anderson Darling estimation (ADE), right tail Anderson Darling estimation (RTADE), and the left tail Anderson Darling estimation (LTADE) method. These methods are used in the estimation process of the

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[1] Department of Mathematical Sciences, Ball State University, Muncie, IN, USA.
Email: mali@bsu.edu

[2] Department of Statistics, Mathematics and Insurance, Benha University, Egypt.

[3] Department of Applied, Mathematical and Actuarial Statistics, Faculty of Commerce, Damietta University, Damietta, Egypt.
Email: mohamed_ibrahim@du.edu.eg

* Corresponding author: haitham.yousof@fcom.bu.edu.eg

unknow parameters. Monte Carlo simulation experiments are performed for comparing the performances of the proposed methods of estimation for both small and large samples.

The OBGR survival function (SF) is given by

$$S_{\underline{\Theta}}(z)=\frac{\left[1-(1-e^{-z^2})^{\beta}\right]^{\delta\theta}}{\left\{(1-e^{-z^2})^{\beta\delta}+\left[1-(1-e^{-z^2})^{\beta}\right]^{\delta}\right\}^{\theta}}, \tag{5}$$

where $S_{\underline{\Theta}}(z)=1-F_{\underline{\Theta}}(z)\,|_{(\underline{\Theta}=\delta,\theta,\beta)}$. For $\theta = 1$, the OBGR reduces to the O-F. For $\delta = 1$, the OBGR reduces to the PRHR-R. The PDF corresponding to (5) is given by

$$f_{\underline{\Theta}}(z)=2\delta\theta\beta z e^{-z^2}\,\frac{(1-e^{-z^2})^{\beta\delta-1}\left[1-(1-e^{-z^2})^{\beta}\right]^{\delta\theta-1}}{\left\{(1-e^{-z^2})^{\beta\delta}+\left[1-(1-e^{-z^2})^{\beta}\right]^{\delta}\right\}^{1+\theta}}. \tag{6}$$

The hazard rate function (HRF) for the new model can be obtained from $f_{\underline{\Theta}}(z)/S_{\underline{\Theta}}(z)$. The asymptotics of the CDF, PDF and hazard rate function (HRF) as $z \to 0$ are given by

$$F_{\underline{\Theta}}(z)\,|_{(z\to 0)}\sim\theta(1-e^{-z^2})^{\beta\delta},\; f_{\underline{\Theta}}(z)\,|_{(z\to 0)}\sim 2\delta\theta\beta z e^{-z^2}(1-e^{-z^2})^{\beta\delta-1},$$

and

$$h_{\underline{\Theta}}(z)\,|_{(z\to 0)}\sim 2\delta\theta\beta z e^{-z^2}(1-e^{-z^2})^{\beta\delta-1}.$$

The asymptotics of CDF, PDF and HRF as $z \to \infty$ are given by

$$1-F_{\underline{\Theta}}(z)\,|_{(z\to\infty)}\sim\delta^{\theta}\left[1-(1-e^{-z^2})^{\beta}\right]^{\theta},$$

$$f_{\underline{\Theta}}(z)\,|_{(z\to\infty)}\sim\frac{2\delta^{\theta}\theta\beta z e^{-z^2}(1-e^{-z^2})^{\beta-1}}{\left[1-(1-e^{-z^2})^{\beta}\right]^{1-\theta}},$$

and

$$h_{\underline{\Theta}}(z)\,|_{(z\to\infty)}\sim\frac{2\theta\beta z e^{-z^2}(1-e^{-z^2})^{\beta-1}}{1-(1-e^{-z^2})^{\beta}}.$$

Figure 1 gives some plots of the OBGR PDF for selected parameter values. Figure 2 gives some plots of the OBGR HRF for selected parameter values.

Based on Figure 1, the new PDF can be "right skewed" with "bimodal" and "unimodal" shapes. Based on Figure 2, the new HRF can be "increasing", " **U**-shape or (bathtub)", "**J**-shape", "upside-down-increasing", "decreasing", "upside-down" or "increasing-constant-increasing".

For simulation of this new model, we obtain the quantile function (QF) of Z (by inverting the CDF), say $z_u = F^{-1}(u)$, as

$$z_u=\left(-\ln\left\{1-\left[\frac{u(\delta,\theta)}{(1-u)^{\frac{1}{\delta\theta}}+u(\delta,\theta)}\right]^{\frac{1}{\beta}}\right\}\right)^{\frac{1}{2}}, \tag{7}$$

where $u(\delta,\theta)=\left[1-(1-u)^{\frac{1}{\theta}}\right]^{\frac{1}{\delta}}$. Equation (7) is used for simulating the new model.

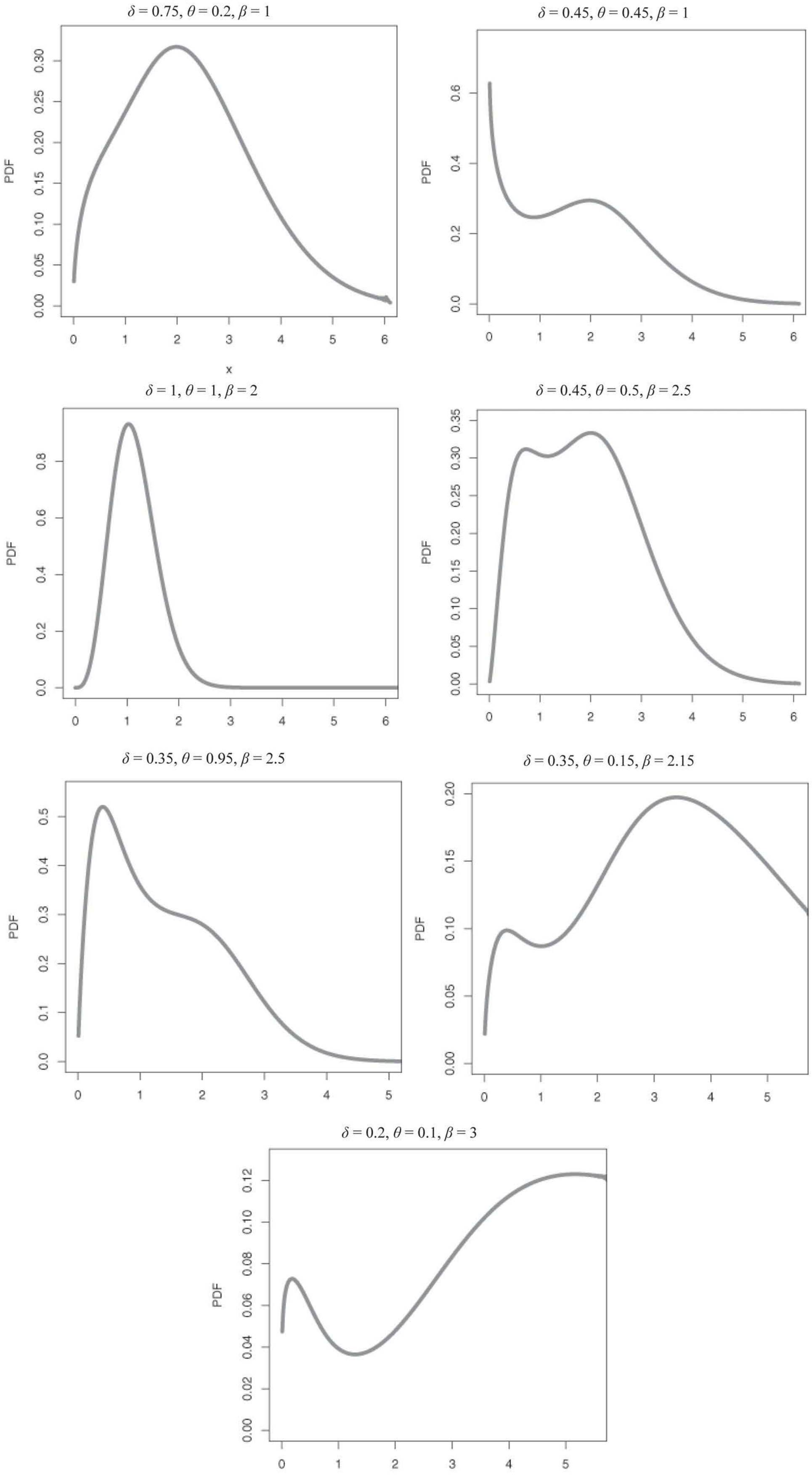
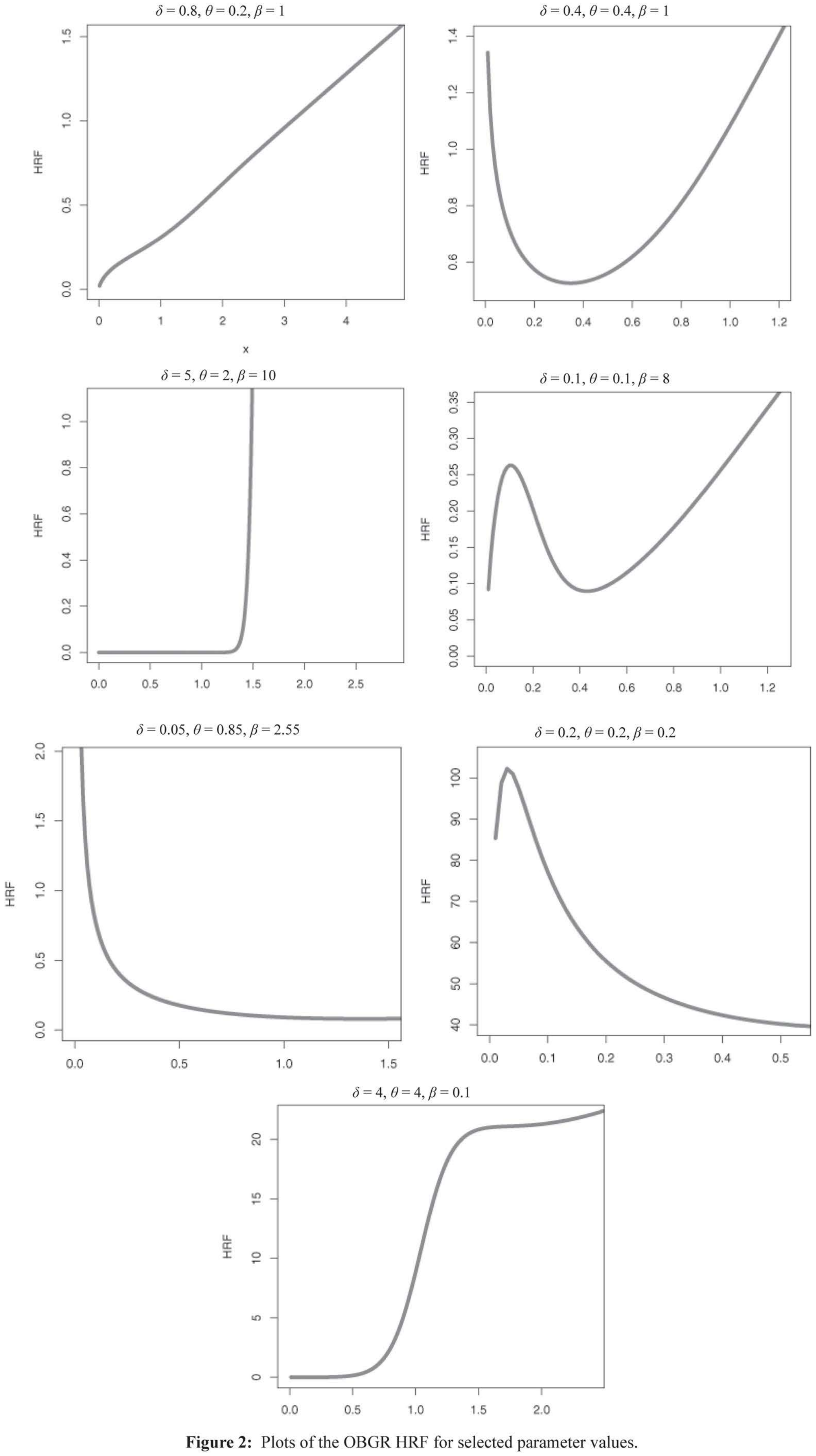


Figure 1: Plots of the OBGR PDF for selected parameter values.



$\delta = 0.8, \theta = 0.2, \beta = 1$

$\delta = 0.4, \theta = 0.4, \beta = 1$

$\delta = 5, \theta = 2, \beta = 10$

$\delta = 0.1, \theta = 0.1, \beta = 8$

$\delta = 0.05, \theta = 0.85, \beta = 2.55$

$\delta = 0.2, \theta = 0.2, \beta = 0.2$

$\delta = 4, \theta = 4, \beta = 0.1$

Figure 2: Plots of the OBGR HRF for selected parameter values.

2. Simple Type Copula

In this section, we derive some new bivariate OBGR (BvOBGR) and Multivariate OBGR (MvOBGR) types. However, future works may be allocated to study these new models. For more details about Copula and distributions see Mansour et al. (2020a,b,c).

2.2 BvOBGR Type via FGM Copula

Consider the joint CDF of the FGM family (see Morgenstern (1956), Gumbel (1960), Farlie (1960), Gumbel (1961), Johnson and Kotz (1975) and Johnson and Kotz (1977))

$$C_{\boldsymbol{\Delta}}(w,\varpi)=w\varpi(1+\boldsymbol{\Delta}\overline{w\varpi}),$$

where the continuous marginal function $w=1-\overline{w}\in[0,1]$, $\varpi=1-\overline{\varpi}\in[0,1]$, $\boldsymbol{\Delta}\in[-1,1]$ is a dependence parameter and for every $C_{\boldsymbol{\Delta}}(w,0)=C_{\boldsymbol{\Delta}}(0,\varpi)=0\,|_{(w,\,\varpi\,\in\,[0,1])}$ which is "grounded minimum" and $C_{\boldsymbol{\Delta}}(w,1)=w$ and $C_{\boldsymbol{\Delta}}(1,\varpi)=\varpi$ which is "grounded maximum". Then, setting $\overline{w}=\overline{w}_{\underline{\Theta}_1}|_{\underline{\Theta}_1=\delta_1,\theta_1,\beta>0}$, and $\overline{\varpi}=\overline{\varpi}_{\underline{\Theta}_2}|_{\underline{\Theta}_2=\delta_2,\theta_2,a>0}$, we have

$$F(z_1,z_2)=\left\{1-\frac{\left(1-\boldsymbol{\varrho}_{z_1}^{\beta}\right)^{\delta_1\theta_1}}{\left[\boldsymbol{\varrho}_{z_1}^{\beta\delta_1}+\left(1-\boldsymbol{\varrho}_{z_1}^{\beta}\right)^{\delta_1}\right]^{\theta_1}}\right\}\left\{1-\frac{\left(1-\boldsymbol{\varrho}_{z_2}^{a}\right)^{\delta_2\theta_2}}{\left[\boldsymbol{\varrho}_{z_2}^{a\delta}+\left(1-\boldsymbol{\varrho}_{z_2}^{a}\right)^{\delta_2}\right]^{\theta_2}}\right\}$$

$$\times\left(1+\boldsymbol{\Delta}\left\{\frac{\left(1-\boldsymbol{\varrho}_{z_1}^{\beta}\right)^{\delta_1\theta_1}\left(1-\boldsymbol{\varrho}_{z_2}^{a}\right)^{\delta_2\theta_2}}{\left[\boldsymbol{\varrho}_{z_1}^{\beta\delta_1}+\left(1-\boldsymbol{\varrho}_{z_1}^{\beta}\right)^{\delta_1}\right]^{\theta_1}\left[\boldsymbol{\varrho}_{z_2}^{a\delta_2}+\left(1-\boldsymbol{\varrho}_{z_2}^{a}\right)^{\delta_2}\right]^{\theta_2}}\right\}\right),$$

where $F(z_1,z_2)=C(F_{\underline{\Theta}_1}(z_1),F_{\underline{\Theta}_2}(z_2))$ and $\boldsymbol{\varrho}_{y}^{\beta}=(1-e^{-y^2})^{\beta}$. The joint PDF can be derived from

$$C_{\boldsymbol{\Delta}}(w,\varpi)=1+\boldsymbol{\Delta}w^{*}\varpi^{*}|_{(w^{*}=1-2w\text{ and }\varpi^{*}=1-2\varpi)},$$

or from

$$f(z_1,z_2)=f_{\underline{\Theta}_1}(z_1)f_{\underline{\Theta}_2}(z_2)C(F_{\underline{\Theta}_1}(z_1),F_{\underline{\Theta}_2}(z_2)).$$

2.3 BvOBGR Type via Modified FGM Copula

Consider the following modified FGM copula defined as (see Rodriguez-Lallena and Ubeda-Flores (2004)) $C_{\boldsymbol{\Delta}}(u,v)=uv+\boldsymbol{\Delta}\widetilde{\boldsymbol{\Phi}(u)}\,\widetilde{\boldsymbol{\psi}(v)}$, $\widetilde{\boldsymbol{\Phi}(u)}=u\overline{\boldsymbol{\Phi}(u)}$ and $\widetilde{\boldsymbol{\psi}(v)}=v\overline{\boldsymbol{\psi}(v)}$, where $\boldsymbol{\Phi}(u)$ and $\boldsymbol{\psi}(v)$ are two absolutely continuous functions on (0,1) with the following conditions:

1-The boundary condition:

$$\boldsymbol{\Phi}(0)=\boldsymbol{\Phi}(1)=\boldsymbol{\psi}(0)=\boldsymbol{\psi}(1)=0.$$

2-Let

$$\alpha=inf\left\{\frac{\partial}{\partial u}\widetilde{\boldsymbol{\Phi}(u)}:A_1\right\}<0,\beta=sup\left\{\frac{\partial}{\partial u}\widetilde{\boldsymbol{\Phi}(u)}:A_1\right\}<0,$$

$$\xi=inf\left\{\frac{\partial}{\partial v}\widetilde{\boldsymbol{\psi}(v)}:A_2\right\}>0,\eta=sup\left\{\frac{\partial}{\partial v}\widetilde{\boldsymbol{\psi}(v)}:A_2\right\}>0.$$

Then, $min(\alpha\beta,\,\xi\eta)\geq 1$, where

$$\frac{\partial}{\partial u}\widetilde{\boldsymbol{\Phi}(u)}=\boldsymbol{\Phi}(u)+u\frac{\partial}{\partial u}\boldsymbol{\Phi}(u)$$

$$A_1=\left\{u\in(0,1):\frac{\partial}{\partial u}\widetilde{\boldsymbol{\Phi}(u)}\quad\text{exists}\right\},$$

and

$$A_2 = \left\{ v \in (0,1) : \frac{\partial}{\partial v} \widetilde{\boldsymbol{\psi}(v)} \quad \text{exists} \right\}.$$

2.3.1 BvOBGR-FGM (Type I)

Here, we consider the following functional form for both $\boldsymbol{\Phi}(u)$ and $\boldsymbol{\psi}(v)$ where $\widetilde{\boldsymbol{\Phi}(u)} = u \dfrac{\left(1-\boldsymbol{\varrho}_u^{\beta}\right)^{\delta_1\theta_1}}{\left[\boldsymbol{\varrho}_u^{\beta\delta_1} + \left(1-\boldsymbol{\varrho}_u^{\beta}\right)^{\delta_1}\right]^{\theta_1}} |_{\underline{\Theta}_1>0}$, and $\widetilde{\boldsymbol{\psi}(v)} = v \dfrac{\left(1-\boldsymbol{\varrho}_v^{a}\right)^{\delta_2\theta_2}}{\left[\boldsymbol{\varrho}_v^{a\delta_2} + \left(1-\boldsymbol{\varrho}_v^{a}\right)^{\delta_2}\right]^{\theta_2}} |_{\underline{\Theta}_2>0}$. Then using $C_{\boldsymbol{\Delta}}(u,v) = uv + \boldsymbol{\Delta}\widetilde{\boldsymbol{\Phi}(u)}\,\widetilde{\boldsymbol{\psi}(v)}$, the BvOBGR-FGM (Type I) can be obtained.

2.3.2 OBGR-FGM (Type II)

Consider the following functional form for both $\Phi(u)$ and $\psi(v)$ which satisfy all the conditions stated earlier where $\boldsymbol{\Phi}(u)|_{(\Delta_1>0)} = u^{\Delta_1}(1-u)^{1-\Delta_1}$ and $\boldsymbol{\psi}(v)|_{(\Delta_2>0)} = v^{\Delta_2}(1-v)^{1-\Delta_2}$.

The corresponding bivariate copula (henceforth, BvOBGR-FGM **(**Type **II)** copula) can be derived from $C_{\boldsymbol{\Delta},\Delta_1,\Delta_2}(u,v) = uv[1+\boldsymbol{\Delta}u^{\Delta_1}v^{\Delta_2}(1-u)^{1-\Delta_1}(1-v)^{1-\Delta_2}]$.

2.3.3 OBGR-FGM (Type III)

Consider the following functional form for both $\boldsymbol{\Phi}(u)$ and $\boldsymbol{\psi}(v)$ which satisfy all the conditions stated earlier where $\boldsymbol{\Phi}(u) = u\,[log(1+\bar{u})]$ and $\boldsymbol{\psi}(v) = v\,[log(1+\bar{v})]$. In this case, one can also derive a closed form expression for the associated CDF of the BvOBGR-FGM (Type **III**).

2.3.4 OBGR-FGM (Type IV)

Using Ghosh and Ray (2016) the CDF of the BvOBGR-FGM (Type **IV**) model can be derived from $C(u, v) = uF^{-1}(v) + vF^{-1}(u) - F^{-1}(u)\,F^{-1}(v)$ where $F^{-1}(u)$ and $F^{-1}(v)$ are derived before.

2.3 BvOBGR Type via Clayton Copula

The Clayton Copula can be considered as $C(v_1,v_2) = (v_1^{-\Delta} + v_2^{-\Delta} - 1)^{-\frac{1}{\Delta}}|_{\Delta\in[0,\infty]}$. Let us assume that $T \sim$ OBGR $(\delta_1, \theta_1, \beta)$ and $X \sim$ OBGR (δ_2, θ_2, a). Then, setting $v_1 = v(t)|_{\underline{\Theta}_1>0}$ and $v_2 = v(x)|_{\underline{\Theta}_2>0}$, the BvOBGR type distribution can be derived from $F(t, x) = C(F_{\underline{\Theta}_1}(t), F_{\underline{\Theta}_2}(x))$.

2.4 BvOBGR Type via Renyi's Entropy

Consider the theorm of Pougaza and Djafari (2011) where $R(w, \varpi) = z_2w + z_1\varpi - z_1z_2$. Then, the associated BvOBGR will be

$$R(z_1,z_2)|_{(\alpha=\alpha_1=\alpha_2)} = R(F_{\underline{\Theta}_1}(z_1), F_{\underline{\Theta}_2}(z_1)) = -z_1z_2$$
$$+ z_2\left\{1-\frac{\left(1-\boldsymbol{\varrho}_{z_1}^{\beta}\right)^{\delta_1\theta_1}}{\left[\boldsymbol{\varrho}_{z_1}^{\beta\delta_1} + \left(1-\boldsymbol{\varrho}_{z_1}^{\beta}\right)^{\delta_1}\right]^{\theta_1}}\right\} + z_1\left\{1-\frac{\left(1-\boldsymbol{\varrho}_{z_2}^{a}\right)^{\delta_2\theta_2}}{\left[\boldsymbol{\varrho}_{z_2}^{a\delta} + \left(1-\boldsymbol{\varrho}_{z_2}^{a}\right)^{\delta_2}\right]^{\theta_2}}\right\}.$$

A straightforward Multivariate OBGR $\hbar$-dimensional extension can be derived from

$$H(\varpi_i) = \left[\sum_{i=1}^{\hbar} \varpi_i^{-\Delta} + 1 - \hbar\right]^{-\frac{1}{\Delta}}.$$

3. Mathematical Properties

3.1 Useful Representations

Due to Alizadeh et al. (2016), the PDF in (6) can be expressed as

$$f(z)=\sum_{k=0}^{\infty}\varOmega_k \pi_{\beta^*}(z)\,|_{(\beta=\beta(1+k))}, \tag{8}$$

where

$$\varOmega_k=\frac{\delta\theta}{1+k}\sum_{j,i=0}^{\infty}\sum_{l=k}^{\infty}(-1)^{i+k+l}\binom{-(1+\theta)}{j}\binom{-[\delta(1+j)+1]}{i}\binom{\delta(1+j)+i+1}{l}\binom{l}{k},$$

and $\pi_\beta(z)$ is the PDF of the EW model with power parameter β. By integrating Equation (8), the CDF of Z becomes

$$F(z)=\sum_{k=0}^{\infty}\varOmega_k H_{\beta^*}(z), \tag{9}$$

where $H_{\beta^*}(z)$ is the CDF of the EW distribution with power parameter β^*.

3.2 Moments and Incomplete Moments

The r^{th} ordinary moment of Z is given by $\mu_r'=E(Z^r)=\int_{-\infty}^{\infty}z^r f(z)dz$. Then we obtain

$$\mu_r'|_{(r>-2)}=\varGamma\left(1+\frac{r}{2}\right)\sum_{k,h=0}^{\infty}\varOmega_{k,h}^{(r,\beta^*)}, \tag{10}$$

where $\varOmega_{k,h}^{(r,\beta^*)}=\varOmega_k\dfrac{\beta^*(-1)^h}{(h+1)^{(r+2)/2}}\dbinom{\beta^*-1}{h}$ and $\varGamma(1+\psi_1)|_{(\psi_1\in R^+)}=\prod_{r=0}^{\psi_1-1}(\psi_1-r)$, where $E(Z)=\mu_1'$ is the mean of Z. The variance (V(Z)), skewness (S(Z)) and kurtosis (K(Z)) can be derived easily using the well-known relationships. The r^{th} incomplete moment, say $I_r(\tau)$, of Z can be expressed, from (9), as

$$I_r(\tau)=\int_{-\infty}^{\tau}z^r f(z)dz=\sum_{k=0}^{\infty}\varOmega_k\int_{-\infty}^{\tau}z^r\pi_{\beta^*}(z)dz.$$

Then

$$I_r(\tau)|_{(r>-2)}=\gamma\left(1+\frac{r}{2},t^2\right)\sum_{k,h=0}^{\infty}\varOmega_{k,h}^{(r,\beta^*)},$$

where $\gamma(\psi_1,\psi_2)$ is the incomplete gamma function.

$$\gamma(\psi_1,\psi_2)=\int_0^{\psi_2}z^{\psi_1-1}e^{-z}dz=\frac{\psi_2^{\psi_1}}{\psi_1}\{1F_1[\psi_1;\psi_1+1;-\psi_2]\}=\sum_{k=0}^{\infty}\frac{(-1)^k}{k!(\psi_1+k)}\psi_2^{\psi_1+k},$$

and $1F_1[\cdot,\cdot,\cdot]$ is a confluent hypergeometric function. The first incomplete moment given by

$$I_1(\tau)=\gamma\left(\frac{3}{2},t^2\right)\sum_{k,h=0}^{\infty}\varOmega_{k,h}^{(1,\beta^*)}.$$

The dispersion index (DisIx) or the variance to mean ratio is a measure used to quantify whether a set of observed occurrences are clustered or dispersed compared to a standard statistical model. A numerical analysis for the DisIx (Z) for the new OBGR is presented in Table 2 with useful comments.

3.3 Moment Generating Function (MGF)

The MGF of Z can be derived from Equation (8) as

$$M_z(\tau)=\sum_{k=0}^{\infty} v_k M_{\beta^*}(\tau),$$

where $M_\beta^*(\tau)$ is the MGF of the GR model, then

$$M_z(\tau)|_{(r>-2)}=\sum_{r=0}^{\infty}\sum_{k,h=0}^{\infty}\frac{\tau^r}{r!}\Gamma\left(1+\frac{r}{2}\right)\Omega_{k,h}^{(r,\beta^*)}.$$

3.4 *Residual Life and Reversed Residual Life Functions*

The r^{th} moment of the residual life $A_r(\tau) = E[(Z-\tau)^r|_{z>\tau,\, r=1,2,\ldots}]$. The r^{th} moment of the residual life of Z is given by

$$A_r(\tau)=\frac{1}{1-F(\tau)}\int_\tau^\infty (Z-\tau)^r\,dF(z).$$

Therefore,

$$A_r(\tau)=\frac{1}{1-F(\tau)}\sum_{k,h=0}^{\infty} a_{k,h}^{(r,\beta^*)}\Gamma\left(1+\frac{r}{2},t^2\right)|_{(r>-2)},$$

where $a_{k,h}^{(r,\beta^*)}=\Omega_k\sum_{m=0}^{r}\binom{r}{m}(-\tau)^{r-m},\Gamma(\psi_1,r)|_{r>0}=\int_r^\infty z^{\psi_1-1}e^{-z}dz$ and $\Gamma(\psi_1,r)=\Gamma(\psi_1)-\gamma(\psi_1,r)$.

The r^{th} moment of the reversed residual life, say $Z_r(\tau)=E[(\tau-Z)^r|_{z\le\tau,\tau>0\,\text{and}\,r=1,2,\ldots}]$ uniquely determines $F(z)$. Then, we obtain $Z_r(\tau)=\frac{1}{F(\tau)}\int_0^\tau(\tau-Z)^r\,dF(z)$. Then, the r^{th} moment of the reversed residual life of Z becomes

$$Z_r(\tau)=\frac{1}{F(\tau)}\sum_{k,h=0}^{\infty} b_{k,h}^{(r,\beta^*)}\gamma\left(1+\frac{r}{2},t^2\right)|_{(r>-2)},$$

where

$$b_{k,h}^{(r,\beta^*)}=\Omega_k\sum_{m=0}^{r}\binom{r}{m}\tau^{m-r}(-1)^m.$$

4. Numerical Analysis

Table 1 gives some numerical calculations for the mean, V(Z), S(Z), K(Z) and DisIx(Z). Based on Table 1, we note that the skewness of the OBGR model can be positive and negative as well. The spread for the OBGR kurtosis is much larger ranging from –16.774 to 75224271. The DisIx(Z) can be "between 0 and 1" and "more than 1".

5. Estimation Methods

In this Section, we briefly describe and consider different classical estimation methods namely, the MLE method, CVM method, OLS method, WLSE method, ADE method, RTADE method, left tail LTADE. All these methods are discussed in the statistical literature with more details. In this work, we may ignore some of its derivation details for avoiding the repetition.

5.1 *The MLE Method*

Let $Z_1, Z_2, \ldots, Z_m$ be any RS from the new OBGR. The log likelihood function $\left(\ell_{[\Theta]}^{(m)}\right)$ for $\underline{\Theta}$ may be expressed as

$$\ell_{[\Theta]}^{(m)}=m\,log(2\delta\theta\beta z)-\sum_{k=1}^{m}z_{[m,k]}^2+(\beta\delta-1)\sum_{k=1}^{m}log\left(1-e^{-z_{[m,k]}^2}\right)$$

$$+(\delta\theta-1)\sum_{k=1}^{m}log\left[1-\left(1-e^{-z_{[m,k]}^2}\right)^\beta\right]$$

$$-(1+\theta)\sum_{k=1}^{m}log\left\{\left(1-e^{-z_{[m,k]}^2}\right)^{\beta\delta}+\left[1-\left(1-e^{-z_{[m,k]}^2}\right)^\beta\right]^\delta\right\}.$$

Table 1: Mean, variance, skewness, kurtosis and dispersion index.

δ	θ	β	E(Z)	V(Z)	S(Z)	K(Z)	DisIx(Z)
0.1	10	2	0.017204	0.00566576	14.73350	327.3479	0.3293370
0.5			0.301815	0.03094014	1.029830	4.555105	0.1025136
1			0.551176	0.02997869	0.159010	2.853232	0.0543905
5			0.961227	0.00418211	–0.809210	4.195831	0.0043508
10			1.032716	0.00118227	–0.940050	4.706613	0.0011448
20			1.069991	0.00031101	–0.993379	4.940258	0.0002907
30			1.082614	0.00013998	–0.774423	–16.77394	0.0001293
40			1.088960	7.9560×10^{-5}	–1.093857	14.74018	7.31×10^{-5}
50			1.092778	5.1102×10^{-5}	–1.026987	6.243116	4.68×10^{-5}
60			1.095328	3.5584×10^{-5}	–1.022265	5.143180	3.25×10^{-5}
100			2.490×10^{-7}	2.6199×10^{-7}	2055.689	42258570	1.052221
2.5	0.5	5	1.567618	0.05841878	0.8909679	4.746629	0.037265960
	1		1.436109	0.02756367	0.3076453	3.891405	0.019193290
	5		1.250278	0.01171343	–0.4948790	3.503164	0.009368659
	10		1.189912	0.00969201	–0.5873196	3.568369	0.008145148
	25		1.118369	0.00790280	–0.6376401	3.620711	0.007066365
	50		1.068920	0.00687203	–0.6543228	3.646666	0.006428947
	100		1.022832	0.00600896	–0.6648330	3.670132	0.005874830
	200		0.979754	0.00527184	–0.6737164	3.694863	0.005380779
	500		0.926985	0.00445288	–0.6858751	3.731091	0.004803618
5	5	0.1	0.012519	7.00243×10^{-5}	0.7876379	6.968286	0.005593269
		0.5	0.418374	0.005015261	–0.4656010	3.404089	0.011987510
		1	0.713840	0.005464616	–0.6484830	3.864594	0.007655137
		5	1.335607	0.003546402	–0.7279617	4.128052	0.002655274
		10	1.559558	0.002855629	–0.7220810	4.117927	0.001831051
		50	2.001435	0.001860090	–0.6989719	4.054250	0.000929378
		100	2.001435	0.001860090	–0.6989719	4.054250	0.000929378
		500	2.510104	0.001197058	–0.6745922	3.987213	0.000476896
		1000	$3.073 \times e^{-8}$	7.10490×10^{-8}	8673.1930	75224271	2.311848000

Following the normal routine of parameter estimation for the MLE of δ, θ and β we differentiate $\ell^{(m)}_{[\Theta]}$ with respect to δ, θ and β to obtain the score vector $\left(\frac{\partial\ell^{(m)}_{[\Theta]}}{\partial\delta}, \frac{\partial\ell^{(m)}_{[\Theta]}}{\partial\theta}, \frac{\partial\ell^{(m)}_{[\Theta]}}{\partial\beta}\right)^T$ as follows

$$U_{(\delta)} = \frac{\partial}{\partial\delta}\ell^{(m)}_{[\Theta]},\ U_{(\theta)} = \frac{\partial}{\partial\theta}\ell^{(m)}_{[\Theta]},\ U_{(c)} = \frac{\partial}{\partial\beta}\ell^{(m)}_{[\Theta]}.$$

Setting the nonlinear system of equations $U_{(\delta)} = U_{(\theta)} = U_{(\beta)} = 0$ and solving them simultaneously yields the MLE of $\Theta = (\delta, \theta, \beta)^T$. These equations cannot be solved analytically. So, statistical software can be used to solve them numerically using iterative methods such as the Newton-Raphson type algorithms.

5.2 The CVME Method

The CVME of the parameters δ, θ and β are obtained via minimizing the following expression with respect to (w.r.t) the parameters δ, θ and β respectively, where

$$CVME_{(\Theta)} = \frac{1}{12}m^{-1} + \sum_{k=1}^{m}\left[F_{\delta,\theta,\beta}(z_{[m,k]}) - c_{(k,m)}\right]^2,$$

and $c_{(k,\,m)} = [(2k-1)/2m]$ and

$$CVME_{(\underline{\Theta})} = \sum_{k=1}^{m}\left(1-\frac{\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta\theta}}{\left\{\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta\delta}+\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta}\right\}^{\theta}}-c_{(k,m)}\right)^{2}.$$

The CVME of the parameters δ, θ and β are obtained by solving the following non-linear equations

$$\sum_{k=1}^{m}\left(1-\frac{\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta\theta}}{\left\{\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta\delta}+\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta}\right\}^{\theta}}-c_{(k,m)}\right)\nabla_{(\delta)}\left(z_{[m,k]};\underline{\Theta}\right)=0,$$

$$\sum_{k=1}^{m}\left(1-\frac{\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta\theta}}{\left\{\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta\delta}+\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta}\right\}^{\theta}}-c_{(k,m)}\right)\nabla_{(\theta)}\left(z_{[m,k]};\underline{\Theta}\right)=0,$$

and

$$\sum_{k=1}^{m}\left(1-\frac{\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta\theta}}{\left\{\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta\delta}+\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta}\right\}^{\theta}}-c_{(k,m)}\right)\nabla_{(\beta)}\left(z_{[m,k]};\underline{\Theta}\right)=0,$$

where

$$\nabla_{(\delta)}(z_{[m,k]};\delta,\theta,\beta)=\partial F_{\delta,\theta,\beta}\left(z_{[m,k]}\right)/\partial\delta,$$

$$\nabla_{(\theta)}(z_{[m,k]};\delta,\theta,\beta)=\partial F_{\delta,\theta,\beta}\left(z_{[m,k]}\right)/\partial\theta,$$

and

$$\nabla_{(\beta)}(z_{[m,k]};\delta,\theta,\beta)=\partial F_{\delta,\theta,\beta}\left(z_{[m,k]}\right)/\partial\beta.$$

5.3 The OLSE Method

Let $F_{\underline{\Theta}}(z_{[m,k]})$ denotes the CDF of OBGR model and let $z_1 < z_2 < \cdots < z_m$ be the m ordered RS. The OLSEs are obtained upon minimizing

$$OLSE(\delta,\theta,\beta)=\sum_{k=1}^{m}\left[F_{\underline{\Theta}}(z_{[m,k]})-b_{(k,m)}\right]^{2}.$$

Then, we have

$$OLSE(\delta,\theta,\beta)=\sum_{k=1}^{m}\left(1-\frac{\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta\theta}}{\left\{\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta\delta}+\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta}\right\}^{\theta}}-b_{(k,m)}\right)^{2},$$

where $b_{(k,m)}=\dfrac{k}{m+1}$. The LSEs are obtained via solving the following non-linear equations

$$0=\sum_{k=1}^{m}\left(1-\frac{\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta\theta}}{\left\{\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta\delta}+\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta}\right\}^{\theta}}-b_{(k,m)}\right)\nabla_{(\delta)}\left(z_{[m,k]};\underline{\Theta}\right),$$

$$0=\sum_{k=1}^{m}\left(1-\frac{\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta\theta}}{\left\{\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta\delta}+\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta}\right\}^{\theta}}-b_{(k,m)}\right)\nabla_{(\theta)}\left(z_{[m,k]};\underline{\Theta}\right),$$

and

$$0=\sum_{k=1}^{m}\left(1-\frac{\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta\theta}}{\left\{\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta\delta}+\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta}\right\}^{\theta}}-b_{(k,m)}\right)\nabla_{(\beta)}\left(z_{[m,k]};\underline{\Theta}\right),$$

where $\nabla_{(\delta)}(z_{[m,k]};\underline{\Theta})$, $\nabla_{(\theta)}(z_{[m,k]};\underline{\Theta})$ and $\nabla_{(\beta)}(z_{[m,k]};\underline{\Theta})$ are defined before.

5.4 The WLSE Method

The WLSE are obtained by minimizing the function **WLSE** (δ, θ, β) w.r.t δ, θ and β where

$$WLSE(\delta,\theta,\beta)=\sum_{k=1}^{m}\omega_{(k,m)}\left[F_{\delta,\theta,\beta}(z_{[m,k]})-b_{(k,m)}\right]^{2},$$

and

$$\omega_{(k,m)}=[(1+m)^{2}(2+m)]/[k(1+m-k)].$$

The WLSEs are obtained by solving

$$0=\sum_{k=1}^{m}\left(1-\frac{\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta\theta}}{\left\{\begin{array}{c}\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta\delta}\\+\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta}\end{array}\right\}^{\theta}}-b_{(k,m)}\right)\omega_{(k,m)}\nabla_{(\delta)}\left(z_{[m,k]};\underline{\Theta}\right),$$

$$0=\sum \omega_{(k,m)}\left(1-\frac{\left[1-\left(1-\quad{}_{[m,k]}\right)\quad\right]^{\delta\theta}}{\left\{\begin{array}{c}\left(\qquad{}_{[m,k]}\right)^{\beta\delta}\\ +\left[1-\left(1-\quad{}_{[m,k]}\right)\quad\right]\end{array}\right\}}-b_{(k,m)}\right)\nabla_{(\ )}\left(z_{[m,k]};\underline{\Theta}\right),$$

and

$$0=\sum_{k=1}^{m}\omega_{(k,m)}\left(1-\frac{\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta\theta}}{\left\{\begin{array}{c}\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta\delta}\\ +\left[1-\left(1-e^{-z_{[m,k]}^{2}}\right)^{\beta}\right]^{\delta}\end{array}\right\}^{\theta}}-b_{(k,m)}\right)\nabla_{(\beta)}\left(z_{[m,k]};\underline{\Theta}\right).$$

5.5 *The ADE Method*

The ADE of δ, θ and β are obtained by minimizing the function

$$ADE_{(z_{[m,k]},z_{[-k+1+m\,:\,m]})}(\delta,\theta,\beta)=-m-m^{-1}\sum_{k=1}^{m}(2k-1)\left\{\begin{array}{c}\log F_{\underline{\Theta}}(z_{[m,k]})\\ +\log\left[1-F_{\underline{\Theta}}(z_{[-k+1+m\,:\,m]})\right]\end{array}\right\}.$$

The parameter estimates of δ, θ and β follow by solving the nonlinear equations

$$0=\partial\left[ADE_{(z_{[m,k]},z_{[-k+1+m\,:\,m]})}(\underline{\Theta})\right]/\partial\delta,$$

$$0=\partial\left[ADE_{(z_{[m,k]},z_{[-k+1+m\,:\,m]})}(\underline{\Theta})\right]/\partial\theta,$$

and

$$0=\partial\left[ADE_{(z_{[m,k]},z_{[-k+1+m\,:\,m]})}(\underline{\Theta})\right]/\partial\beta.$$

5.6 *The RTADE Method*

The RTADE of δ, θ and β are obtained by minimizing

$$\mathrm{RTADE}_{(z_{[m,k]},z_{[-k+1+m\,:\,m]})}(\delta,\theta,\beta)=\frac{1}{2}m-2\sum_{k=1}^{m}F_{\underline{\Theta}}(z_{[m,k]})$$

$$-\frac{1}{m}\sum_{k=1}^{m}(2k-1)\left\{\log\left[1-F_{(\delta,\theta,\beta)}(z_{[-k+1+m\,:\,m]})\right]\right\}.$$

The estimates of δ, θ and β are obtained by solving the nonlinear equations

$$0=\partial\left[\mathrm{RTADE}_{(z_{[m,k]},z_{[-k+1+m\,:\,m]})}(\underline{\Theta})\right]/\partial\delta,$$

$$0 = \partial\left[\text{RTADE}_{(z_{[m,k]}, z_{[-k+1+m\,:\,m]})}(\underline{\Theta})\right] / \partial\theta,$$

and

$$0 = \partial\left[\text{RTADE}_{(z_{[m,k]}, z_{[-k+1+m\,:\,m]})}(\underline{\Theta})\right] / \partial\beta.$$

5.7 The LTADE Method

The LTADE of δ, θ and β are obtained by minimizing

$$\text{LTADE}_{(z_{[m,k]})}(\delta,\theta,\beta) = -\frac{3}{2}m + 2\sum_{k=1}^{m} F_{\underline{\Theta}}(z_{[m,k]}) - \frac{1}{m}\sum_{k=1}^{m}(2k-1)\log F_{\underline{\Theta}}(z_{[m,k]}).$$

The parameter estimates of δ, θ and β are obtained by solving the nonlinear equations

$$0 = \partial\left[\text{LTADE}_{(z_{[m,k]})}(\underline{\Theta})\right] / \partial\delta,$$

$$0 = \partial\left[\text{LTADE}_{(z_{[m,k]})}(\underline{\Theta})\right] / \partial\theta,$$

Table 2: Simulation results for parameters $\delta = 1.5$, $\theta = 1.2$, $\beta = 0.6$.

Methods	***m***	**Bias**			**RMSE**			**D**	
		$\boldsymbol{\delta}$	$\boldsymbol{\theta}$	$\boldsymbol{\beta}$	$\boldsymbol{\delta}$	$\boldsymbol{\theta}$	$\boldsymbol{\beta}$	**abs**	**max**
MLE	50	0.01733	0.01462	0.00692	0.18690	0.17700	0.06148	0.00326	0.00604
OLS		0.00587	0.02778	0.02002	0.23111	0.19671	0.06918	0.01881	0.02701
WLS		0.15120	0.03727	0.03172	0.28615	0.17838	0.07225	0.03267	0.05784
CVM		0.01435	0.01284	0.00674	0.23040	0.20309	0.06500	0.00132	0.00228
ADE		0.03671	0.00639	0.00571	0.19604	0.18912	0.06281	0.00318	0.00585
RTADE		0.02503	0.00090	0.00992	0.22661	0.18012	0.07007	0.00553	0.00901
LTADE		0.01352	0.02381	0.00355	0.27197	0.22863	0.06221	0.00301	0.00601
MLE	100	0.01611	0.01197	0.00354	0.12770	0.12313	0.04202	0.00232	0.00364
OLS		0.01121	0.01143	0.00949	0.15661	0.13793	0.04626	0.00887	0.01312
WLS		0.10673	0.01317	0.01794	0.18434	0.12787	0.04852	0.01863	0.03469
CVM		0.00069	0.00884	0.00297	0.15601	0.14051	0.04484	0.00065	0.00102
ADE		0.01159	0.00590	0.00247	0.13343	0.13102	0.04337	0.00074	0.00117
RTADE		0.00235	0.00477	0.00360	0.16239	0.12544	0.04788	0.00130	0.00198
LTADE		0.01434	0.00940	0.00264	0.18054	0.15361	0.04305	0.00198	0.00305
MLE	150	0.00824	0.00894	0.00116	0.10411	0.09707	0.03410	0.00139	0.00276
OLS		0.00556	0.00446	0.00510	0.12838	0.11094	0.03658	0.00442	0.00659
WLS		0.07542	0.00821	0.01268	0.14125	0.10039	0.03842	0.01308	0.02443
CVM		0.00158	0.00910	0.00079	0.12819	0.11273	0.03594	0.00142	0.00206
ADE		0.00918	0.00626	0.00064	0.11079	0.10508	0.03483	0.00116	0.00215
RTADE		0.00311	0.00468	0.00138	0.12951	0.09941	0.03803	0.00063	0.00115
LTADE		0.00409	0.01250	0.00005	0.14916	0.12634	0.03514	0.00244	0.00404
MLE	300	0.00133	0.00034	0.00138	0.07307	0.06743	0.02462	0.00087	0.00138
OLS		0.00022	0.00560	0.00349	0.08828	0.07930	0.02598	0.00344	0.00490
WLS		0.04689	0.00801	0.00946	0.09546	0.07007	0.02757	0.00945	0.01719
CVM		0.00377	0.00115	0.00132	0.08830	0.07963	0.02568	0.00053	0.00094
ADE		0.00740	0.00012	0.00112	0.07778	0.07477	0.02500	0.00076	0.00149
RTADE		0.00188	0.00178	0.00132	0.09320	0.07182	0.02750	0.00019	0.00027
LTADE		0.00213	0.00330	0.00065	0.10257	0.08854	0.02476	0.00041	0.00079

and

$$0 = \partial\left[\mathrm{LTADE}_{(z_{[m,k]})}(\underline{\Theta})\right]/\partial\beta.$$

6. Simulations for Comparing Methods

A numerical simulation is performed to compare the classical estimation methods. The simulation study is based on $N = 1000$ generated data sets from the OBGR version where $m = 50$, 100, 150 and 300 and

	δ	θ	β
I	1.5	1.2	0.6
II	2.5	2.5	0.9
II	2	0.5	0.5

The estimates are compared in terms of their biases, root mean square errors (RMSE). The mean of the absolute difference between the theoretical and the estimates (D-abs) and the maximum absolute difference between the true parameters and estimates (D-max) are also reported.

Tables 2, 3 and 4 give the simulation results. From Tables 2, 3 and 4 we note that the RMSE ($\underline{\Theta}$) tend to zero when m increases which means the incidence of consistency property.

Table 3: Simulation results for parameters $\delta = 2.9$, $\theta = 2.5$, $\beta = 0.9$.

Methods	***m***	**Bias**			**RMSE**			**D**	
		$\boldsymbol{\delta}$	$\boldsymbol{\theta}$	$\boldsymbol{\beta}$	$\boldsymbol{\delta}$	$\boldsymbol{\theta}$	$\boldsymbol{\beta}$	**abs**	**max**
MLE	50	0.03361	0.04490	0.00240	0.27553	0.38545	0.04710	0.00266	0.00429
OLS		0.10336	0.03186	0.00954	0.40371	0.41314	0.04707	0.02093	0.03238
WLS		0.19892	0.05335	0.01547	0.38032	0.37650	0.04860	0.03660	0.05730
CVM		0.03153	0.05360	0.00002	0.37792	0.43211	0.04566	0.00317	0.00649
ADE		0.00082	0.03925	0.00010	0.32346	0.39797	0.04444	0.00390	0.00578
RTADE		0.00215	0.01373	0.00377	0.34235	0.37911	0.04743	0.00222	0.00335
LTADE		0.02252	0.05947	0.00045	0.32158	0.48701	0.04807	0.00359	0.00693
MLE	100	0.02016	0.03802	0.00073	0.19294	0.25899	0.03148	0.00276	0.00536
OLS		0.07096	0.03498	0.00710	0.27542	0.28953	0.03382	0.01622	0.02459
WLS		0.15163	0.03916	0.01146	0.26252	0.26729	0.03589	0.02759	0.04334
CVM		0.03547	0.00702	0.00234	0.26385	0.29312	0.03298	0.00471	0.00804
ADE		0.01809	0.00258	0.00234	0.22553	0.27752	0.03254	0.00348	0.00556
RTADE		0.00854	0.00411	0.00241	0.24893	0.27002	0.03408	0.00252	0.00387
LTADE		0.01410	0.02165	0.00083	0.22100	0.32009	0.03342	0.00113	0.00230
MLE	150	0.01851	0.00960	0.00179	0.15703	0.21778	0.02713	0.00242	0.00421
OLS		0.03715	0.01218	0.00343	0.21363	0.23408	0.02680	0.00765	0.01181
WLS		0.10575	0.01764	0.00693	0.20685	0.21351	0.02842	0.01745	0.02797
CVM		0.01386	0.01598	0.00026	0.20845	0.23736	0.02649	0.00101	0.00175
ADE		0.00421	0.01297	0.00016	0.18401	0.22277	0.02598	0.00079	0.00149
RTADE		0.00619	0.01098	0.00028	0.19702	0.21171	0.02645	0.00140	0.00203
LTADE		0.00950	0.01976	0.00053	0.18210	0.26235	0.02731	0.00111	0.00196
MLE	300	0.00158	0.01471	0.00082	0.10669	0.14459	0.01813	0.00206	0.00305
OLS		0.02204	0.01078	0.00223	0.14725	0.16406	0.01890	0.00506	0.00769
WLS		0.06838	0.01156	0.00459	0.13949	0.15153	0.02017	0.01142	0.01831
CVM		0.01046	0.00317	0.00064	0.14514	0.16481	0.01875	0.00126	0.00220
ADE		0.00445	0.00184	0.00061	0.12726	0.15611	0.01845	0.00077	0.00127
RTADE		0.00181	0.00384	0.00039	0.14730	0.15067	0.01910	0.00020	0.00036
LTADE		0.00587	0.00455	0.00015	0.12671	0.18272	0.01935	0.00063	0.00115

Table 4: Simulation results for parameters $\delta = 2$, $\theta = 0.5$, $\beta = 0.5$.

Methods	m	**Bias**			**RMSE**	**RMSE**	**RMSE**	**D**	
		δ	θ	β	δ	θ	β	**abs**	**max**
MLE	50	0.02848	0.00725	0.00909	0.26069	0.07386	0.05986	0.00385	0.00639
OLS		0.03190	0.00785	0.01864	0.36466	0.08230	0.06854	0.01767	0.02565
WLS		0.12240	0.01222	0.03121	0.31694	0.07352	0.07041	0.01907	0.03656
CVM		0.03009	0.00918	0.00559	0.37646	0.08581	0.06414	0.00428	0.00808
ADE		0.01793	0.00667	0.00349	0.29050	0.07912	0.06067	0.00028	0.00049
RTADE		0.01939	0.00460	0.00765	0.33671	0.07583	0.07181	0.00300	0.00569
LTADE		0.03993	0.01368	0.00282	0.28938	0.09751	0.05895	0.00363	0.00484
MLE	100	0.01160	0.00295	0.00340	0.17582	0.05041	0.03932	0.00152	0.00236
OLS		0.00057	0.00103	0.00659	0.24948	0.05577	0.04361	0.00474	0.00718
WLS		0.10444	0.00436	0.01784	0.22326	0.05027	0.04571	0.01023	0.02105
CVM		0.03179	0.00750	0.00025	0.25542	0.05734	0.04243	0.00570	0.01018
ADE		0.00799	0.00613	0.00015	0.20615	0.05376	0.04056	0.00364	0.00579
RTADE		0.01857	0.00370	0.00400	0.24179	0.05449	0.05137	0.00216	0.00350
LTADE		0.01240	0.00865	0.00018	0.19663	0.06350	0.03884	0.00245	0.00415
MLE	150	0.01175	0.00470	0.00160	0.14364	0.04281	0.03270	0.00226	0.00426
OLS		0.01589	0.00507	0.00871	0.21893	0.04950	0.03863	0.00912	0.01318
WLS		0.07835	0.00589	0.01670	0.18488	0.04465	0.03980	0.00974	0.01974
CVM		0.00479	0.00053	0.00445	0.22075	0.04989	0.03750	0.00232	0.00408
ADE		0.00978	0.00028	0.00376	0.17852	0.04669	0.03573	0.00318	0.00469
RTADE		0.00635	0.00142	0.00256	0.18387	0.04169	0.03987	0.00212	0.00312
LTADE		0.01609	0.00180	0.00265	0.16793	0.05396	0.03312	0.00184	0.00304
MLE	300	0.00130	0.00069	0.00139	0.09969	0.02819	0.02173	0.00132	0.00191
OLS		0.00348	0.00086	0.00272	0.14325	0.03718	0.02559	0.0039	0.00349
WLS		0.05488	0.00133	0.00886	0.12405	0.03001	0.02588	0.00508	0.01030
CVM		0.00686	0.00196	0.00061	0.14412	0.03350	0.02533	0.00113	0.00213
ADE		0.00131	0.00150	0.00035	0.11671	0.03133	0.02421	0.00044	0.00066
RTADE		0.00543	0.00089	0.00207	0.12841	0.02906	0.02736	0.00101	0.00195
LTADE		0.00790	0.00226	0.00039	0.11140	0.03323	0.02318	0.00240	0.00081

Table 5: Comparing estimation methods via an application.

Methods	$\hat{\delta}$	$\hat{\theta}$	$\hat{\beta}$	**CVM***	**AD***
MLE	0.72446	0.18093	1.2458	**0.05531**	**0.55916**
OLS	0.78606	0.17720	3.38609	0.17462	1.47304
WLS	0.81674	0.20699	4.95758	0.26054	2.02331
CVM	0.77836	0.18221	3.39813	0.17096	1.44932
ADE	0.74950	0.17803	2.38404	0.10569	0.99061
RTADE	0.72604	0.19766	3.66245	0.16890	1.44117
LTADE	0.78181	0.15782	1.89213	0.08869	0.85082

7. Modeling Failure Times Data and Comparing Methods

An application to real data set is considered for comparing the estimation methods. The data consist of 84 aircraft windshield observations (see Murthy et al. (2004)). The required computations are carried out using the MATHCAD software. In order to compare the estimation methods, we consider the Cramér-von Mises (CVM) and the Anderson-Darling (AD) statistics. These two statistics are widely used to determine how closely a specific CDF fits the empirical distribution of a given data set. These statistics are given by

$$\mathrm{CVM}^* = \left[\frac{1}{12m} + \sum_{s=1}^{m}\left(z_{\hbar} - \frac{2s-1}{2m}\right)^2\right]\left(1 + \frac{1}{2m}\right),$$

and

$$\mathrm{AD}^{*}=\left(1+\frac{9}{4m^{2}}+\frac{3}{4m}\right)\left\{m+\frac{1}{m}\sum_{s=1}^{m}(2s-1)\,log\left[z_{\hbar}(1-z_{m-s+1})\right]\right\},$$

respectively, where $z_{\hbar}=F(z_s)$ and the z_s's values are the ordered observations. The smaller these statistics are, the better the fit. From Table 6 we conclude that the MLE method is the best method with CVM = 0.05531 and AD = 0.55916. However, all other methods performed well.

Table 6: MLEs and standard errors (SEs) for failure times data set.

Distribution	Estimates (SEs)				
GR (β,α)	1.181876	0.377525			
	(0.17060)	(0.02532)			
OBGR (δ,θ,β)	**0.72167**	**0.1817**	**1.25096**		
	(0.19306)	**(0.0568)**	**(0.5024)**		
OLEW (θ,β,α)	0.15935	0.7322	0.7650		
	(0.3712)	(1.778)	(0.041)		
OLGR (θ,β,α)	1.45406	0.7543	0.2379		
	(0.9018)	(0.2530)	(0.0317)		
GREW (θ,β,α)	0.63684	4.2622	0.5364		
	(0.356)	(1.757)	(0.0997)		
PTLW (θ,β,α)	–5.78175	4.22865	0.65801		
	(1.395)	(1.167)	(0.039)		
MOEW (θ,β,α)	488.899	0.2832	1261.97		
	(189.358)	(0.013)	(351.07)		
GamW (θ,β,α)	2.37697	0.84809	3.5344		
	(0.378)	(0.00053)	(0.665)		
KumW (δ,θ,β,α)	14.4331	0.2041	34.6599	81.8459	
	(27.095)	(0.042)	(17.527)	(52.014)	
WFr (δ,θ,c,α)	630.938	0.3024	416.097	1.1664	
	(697.94)	(0.032)	(232.359)	(0.357)	
Beta-W (δ,θ,β,α)	1.360	0.2981	34.1802	11.496	
	(1.002)	(0.06)	(14.838)	(6.73)	
TrMW (δ,θ,β,α)	0.2722	1	4.6×10^{-6}	0.4685	
	(0.014)	(5.2×10^{-5})	(1.9×10^{-4})	(0.165)	
KumTrW (δ,θ,λ,β,α)	27.7912	0.178	0.4449	29.5253	168.06
	(33.401)	(0.017)	(0.609)	(9.792)	(129.17)
MBW (δ,θ,λ,β,α)	10.1502	0.1632	57.4167	19.3859	2.0043
	(18.697)	(0.019)	(14.063)	(10.019)	(0.662)
MacW (δ,θ,λ,β,α)	1.9401	0.306	17.686	33.6388	16.7211
	(1.011)	(0.045)	(6.222)	(19.994)	(9.722)
TrEGW (δ,θ,λ,β,α)	4.2567	0.1532	0.0978	5.2313	1173.33
	(33.401)	(0.017)	(0.609)	(9.792)	(6.999)

8. Modeling Failure Times Data and Comparing Models

In this section based on the data of Murthy et al. (2004), we provide an application of the OBGR distribution to show empirically its potentiality. The required computations are carried out using the R software.

Here, we shall compare the fits of the OBGR distribution with those of other competitive models, namely, GR, Odd Lindley EW (OLEW) (Silva et al. (2017)), Burr X EW (GREW) (Khalil et al. (2019)), Poisson Topp Leone-W (PTLW), MO extended-W (MOEW) (Ghitany et al. (2005)), Gamma-W (GamW) (Provost et al. (2011)), Kumaraswamy-W (KumW) (Cordeiro et al. (2010)), Beta-W (Lee et al. (2007)), Transmuted modified-W (TrMW) (Khan and King (2013)), W-Fréchet (WFr) (Afify et al. (2016b)), Kumaraswamy transmuted-W (KumTrW) (Afify et al. (2016a)), Modified beta-W (MBW) (Khan (2015)) Mcdonald-W (MacW) (Cordeiro et al. (2014)), transmuted exponentiated generalized W (TrEGW) (Yousof et al. (2015)) distributions. The MLEs and the corresponding standard errors (in parentheses) of the model parameters are given in Table 6. The numerical values of the statistics CVM* and AD* are listed in Table 7. Figure 3 gives the TTT, Q-Q, box, and Estimated HRF (EHRF) plots for failure times data. Figure 4 gives the EPDF, ECDF, Probability-Probability (P-P) and Kaplan-Meier survival plots for the failure times data.

Table 7: CVM and AD statistics.

Distribution	CVM*	AD*
OBGR	**0.0553**	**0.5589**
OLEW	0.0723	0.6086
OLGR	0.0792	0.5910
GR	0.0690	0.6916
GREW	0.0744	0.6420
PTLW	0.1397	1.1939
MOEW	0.3995	4.4477
GamW	0.2553	1.9489
KumW	0.1852	1.5059
WFr	0.2537	1.9574
Beta-W	0.4652	3.2197
TrMW	0.8065	11.205
KumTrW	0.1640	1.3632
MBW	0.4717	3.2656
MacW	0.1986	1.5906
TrEGW	1.0079	6.2332

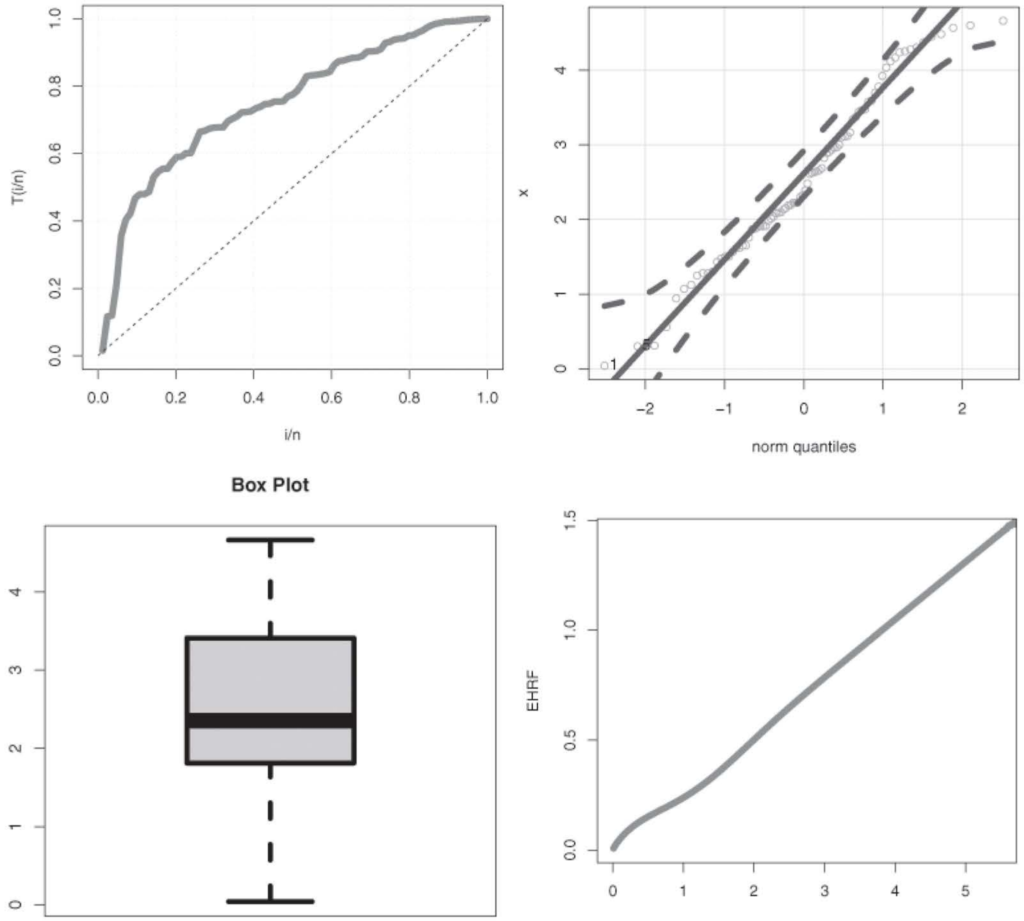


Figure 3: TTT, Q-Q, box, and EHRF plots for failure times data.

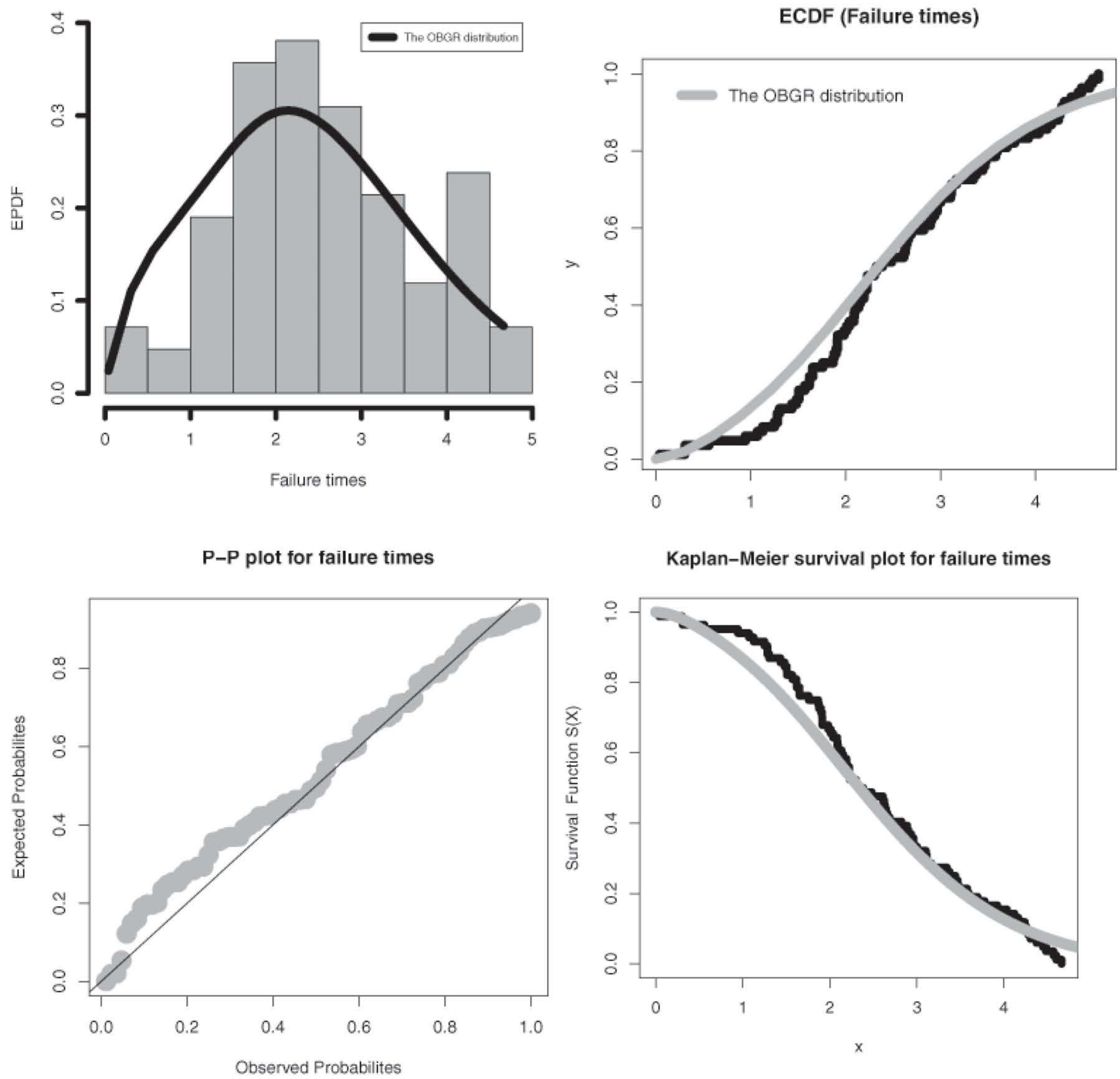


Figure 4: EPDF, ECDF, P-P and Kaplan-Meier plot plots for the failure times data.

Some other extensions of the Rayleigh distribution can also be used in this comparison, but are not limited to Alizadeh et al. (2016), Yousof et al. (2016a,b), Cordeiro et al. (2017a,b), Yousof et al. (2017), Brito et al. (2017), Aryal et al. (2017a,b), Korkmaz et al. (2017), Yousof et al. (2018), Ibrahim and Yousof (2020), Mansour et al. (2020d) and Hamedani et al. (2018 and 2019). Based on the figures in Table 3 we conclude that the new lifetime model provides adequate fits as compared to other W models with small values for CVM and AD. The proposed OBGR lifetime model is much better than the GREW, PTLW, MOEW, GamW, KumW, WFr, Beta-W, TrMW, KumTrW, MBW, MacW, TrEGW models, and a good alternative to these models.

9. Concluding Remarks

We introduced and studied a new version of the Generalized Rayleigh distribution. Some of its properties are derived and numerically analyzed. The usefulness and flexibility of the new distribution are illustrated by means of a real data set. The new PDF can be "right skewed" with "bimodal" and "unimodal" shapes. The new HRF can be "increasing", " **U**-shaped or(bathtub)", "**J**-shaped", "upside-down-increasing", "decreasing", "upside-down" or "increasing-constant-increasing". Many bivariate and multivariate type distributions have also been derived based on FGM Copula, Clayton Copula, modified FGM Copula and Renyi's entropy. We briefly describe different estimation methods namely, the maximum likelihood method, Cramér-von-Mises method, ordinary least square method, weighted least square method, Anderson Darling method, right tail Anderson Darling method, left tail Anderson Darling method which are used in the estimation process. Monte Carlo simulation experiments are performed for comparing the performances of the proposed methods of estimation for both small and large samples.

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Appendix

0.040, 1.866, 2.385, 3.443, 0.301, 1.876, 2.481, 3.467, 0.309, 1.899, 2.610, 3.478, 0.557, 1.911, 2.625, 4.570, 1.652, 2.300, 3.344, 4.602, 1.757, 3.578, 0.943, 1.912, 2.632, 3.595, 1.070, 1.914, 2.646, 3.699, 1.124, 1.981, 2.661, 3.779, 1.248, 2.010, 2.224, 3.117, 4.485, 1.652, 2.229, 3.166, 2.688, 3.924, 1.281, 2.038, 2.823, 4.035, 1.281, 2.085, 2.890, 4.121, 1.303, 2.089, 2.902, 4.167, 1.432, 4.376, 1.615, 2.223, 3.114, 4.449, 1.619, 2.097, 2.934, 4.240, 1.480, 2.135, 2.962, 4.255, 1.505, 2.154, 2.964, 4.278, 1.506, 2.190, 3.000, 4.305, 1.568, 2.194, 3.103, 2.324, 3.376, 4.663.

A New Version of the Generalized Rayleigh Distribution with Copula, Properties, Applications and Different Methods of Estimation

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A New Unrelated Question Model with Two Questions Per Card

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Hybrid of Simple Model and a New Unrelated Question Model for Two Sensitive Characteristics

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Modified Regression Type Estimator by Ingeniously Utilizing Probabilities for more Efficient Results in Randomized Response Sampling

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Ratio and Regression Type Estimators for a New Measure of Coefficient of Dispersion Relative to the Empirical Mode

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Class of Exponential Ratio Type Estimator for Population Mean in Adaptive Cluster Sampling

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An Inventory Model for Substitutable Deteriorating Products under Fuzzy and Cloud Fuzzy Demand Rate

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